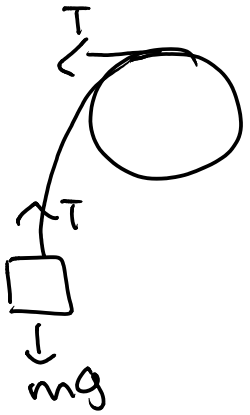
Angular Acceleration of a Falling Object

Wednesday, April 2, 2014 9:22 AM

A wheel of radius R and mass M and moment of inertia I is mounted on a horizontal axle. A light cord is wrapped around the wheel holding an object with mass m. When the wheel is released, the object accelerates downward, the cord unwraps, and the wheel rotates with an angular acceleration. Calculate the angular acceleration of the wheel, the translational acceleration of the object, and the tension in the cord.

Two things going on here
- Model object as particle under net force
- Model wheel as object under net torque



$\tau_{net} = I\alpha$

$\tau_{net} = TR$

↑ force exerted by cord on wheel

$\alpha = \frac{\tau_{net}}{I} = \frac{TR}{I}$

$F_{net} = ma$

$= mg - T$

$ma = mg - T$

$a = mg - T/m$

Note: wheel & object connected (& cord not slipping)

→ so translational acc of object = acc of wheel rim

so α & a are related in the normal way

$a = \alpha R$

$a = R\alpha = R\, TR/I = \frac{mg - T}{m}$

$$T = \frac{mg}{1 + (mR^2/I)}$$

$$a = \frac{g}{1 + (I/mR^2)}$$

$$\alpha = a/R = \frac{g}{R + (I/mR)}$$