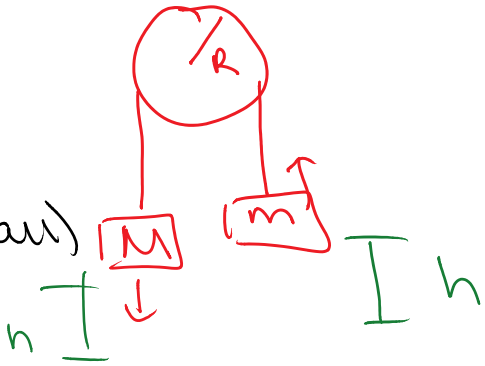
Atwood Machine

Wednesday, April 2, 2014 9:32 AM

Two blocks of mass M and m are connected by a string passing over a pulley. The pulley has radius R and moment of inertia I. The string does not slip and the system is released from rest. Find the translational speeds of the blocks after the block 1 (mass M) descends through a distance h. Find the angular speed of the pulley at this time.



→ pulley rotates about axle

→ can neglect friction in axle (b/c axle small)

so system isolated ⇒ energy is conserved

Energy Conserved

$KE_i + PE_i = KE_f + PE_f$

(set as 0 pt)

$0 + 0 = \frac{1}{2}Mv_f^2 + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + Mgh - mgh$

$\frac{1}{2}Mv_f^2 + \frac{1}{2}mv_f^2 + \frac{1}{2}I\frac{v_f^2}{R^2} = mgh - Mgh$

$\frac{1}{2}(M + m + I/R^2)v_f^2 = mgh - Mgh$

$v_f^2 = \frac{2(m-M)gh}{M + m + I/R^2}$

$$v_f = \sqrt{\frac{2(m-M)gh}{M + m + I/R^2}}$$

$$\omega_f = \frac{v_f}{R} = \frac{1}{R}\sqrt{\frac{2(m-M)gh}{M + m + I/R^2}}$$