Conservation of Momentum Examples

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Example - Stick and Disk Collision

- A disk of mass M traveling at v strikes a stick of mass M/2 of length L lying flat on frictionless ice. The disk strikes at the endpoint of the stick, L/2 from the center. Assume the collision is elastic and the disk does not deviate from its line of motion. Find the translational speed of the disk, stick, and angular speed of the stick after the collision.

What we expect
- The disk keeps moving in the same direction but at a slower speed
- The stick has both translational and rotational motion

Equations to use
- Momentum and Angular momentum (no friction, system isolated)
- Elastic collision means KE is conserved

Three unknowns - speed of disk, speed of stick, angular speed of stick - so three equations

Conservation of Momentum

$p_i = p_f$

$m_d V_{di} = m_d V_{df} + M_s V_s$

$m_d(V_{di} - V_{df}) = M_s V_s$

Conservation of Angular Momentum

$L_i = L_f$

$I\omega = I\omega$

For the disk → it's like a pt mass R distance away from the CM of the stick

$I = MR^2$

$I\omega = MR^2\omega = MR^2 V/R = MVR$

$-MV_{di}R = -MV_{df}R + I\omega$

negative because of direction of motion

$-MR(V_{di} - V_{df}) = I\omega$

Conservation of KE

$KE_{di} = KE_{df} + KE_{sf} + KE_{srf}$

$\frac{1}{2} m_d V_{di}^2 = \frac{1}{2} m_d V_{df}^2 + \frac{1}{2} M_s V_{sf}^2 + \frac{1}{2} I\omega^2$

$m_d(V_{di}^2 - V_{df}^2) = M_s V_s^2 + I\omega^2$

$m_d(V_{di} + V_{df})(V_{di} - V_{df}) = M_s V_s^2 + I\omega^2$

$m_d(V_{di} - V_{df}) = m_s V_s$ → multiply by r

$r m_d(V_{di} - V_{df}) = r m_s V_s$

$+(-rm(V_{di} - V_{df}) = I\omega)$

$$-rm(v_{di}-v_{df}) = I\omega$$

$$0 = I\omega + r m_s v_s$$

$$\omega = -\frac{r m_s v_s}{I}$$

$$\frac{\cancel{m_d}(v_{di}+v_{df})\cancel{(v_{di}-v_{df})}}{\cancel{m_d}\cancel{(v_{di}-v_{df})}} = \frac{m_s v_s^2 + I\omega^2}{m_s v_s}$$

$$v_{di} + v_{df} = v_s + \frac{I\omega^2}{m_s v_s}$$

$$v_{di} + v_{df} = v_s + \frac{I\left(\frac{-r m_s v_s}{I}\right)^2}{m_s v_s}$$

$$v_{di} + v_{df} = v_s + \frac{r^2 m_s v_s}{I}$$

$$v_{di} + v_{df} = v_s\left(1 + \frac{r^2 m}{I}\right)$$

$$v_{di} + \left(v_{di} - \frac{m_s}{m_d} v_s\right) = v_s\left(1 + \frac{r^2 m_s}{I}\right)$$

$$v_s = \frac{2v_{di}}{1 + (m_s/m_d) + r^2 m_s/I} = \frac{2v_{di}}{1 + \frac{M/2}{M} + 1/3} = \frac{2v_{di}}{1 + 1/2 + 1/3} =$$

$$I_{rod} = 1/12\, m_s (2r)^2$$
$$= \frac{1}{12} m_s (4R^2)$$
$$= 1/3\, m_s R^2$$

$$= \frac{2v_{di}}{\frac{11}{6}} = \boxed{\frac{12v_{di}}{11}}$$

$$\omega = -\frac{r m_s v_s}{I} = -\frac{\cancel{r}\,\frac{\cancel{M}}{2}\left(\frac{12v_{di}}{11}\right)}{1/3\,\frac{\cancel{m}}{2} R^{\cancel{2}}}$$

$$= \frac{12v_{di}\cdot 3}{11R} = \frac{36v_{di}}{11L/2} = \boxed{\frac{72v_{di}}{11L}}$$

$$v_{df} = v_{di} - \frac{m_s}{m_d} v_s = v_{di} - \frac{m/2}{m}\left(\frac{12v_{di}}{11}\right)$$

$$= v_{di} - 1/2\left(\frac{12v_{di}}{11}\right)$$

$$= v_{di} - \frac{6}{11} v_{di}$$

$$= \boxed{5/11\, v_{di}}$$