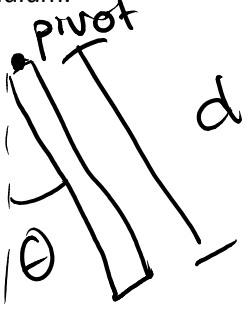
Physical Pendulum

Sunday, April 6, 2014 3:05 PM

A physical pendulum consists of a uniform rod of length d and mass m pivoted at one end. The pendulum is displaced by a small angle and released from rest. Find the period of the pendulum.



$$T = 2\pi\sqrt{\frac{I_{support}}{mgL_{cm}}}$$

$$I = \frac{1}{3}md^2$$

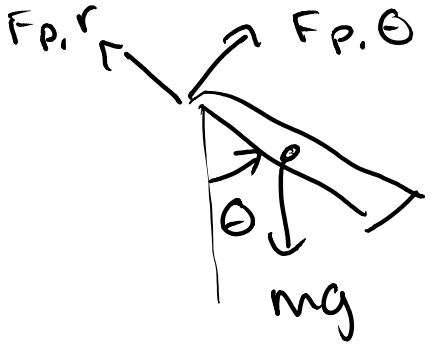
$$L_{cm} = \frac{d}{2}$$

$$T = 2\pi\sqrt{\frac{\frac{1}{3}md^2}{mgd/2}}$$

$$T = 2\pi\sqrt{\frac{\frac{1}{3}d}{g/2}}$$

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$

Calculating with torque



$$\tau_p = r_{cm}mg$$

$$\tau_p = (d/2)mg\sin\theta)$$

$$= -d/2\, mg \sin\theta$$

$$\tau_p = I_p \alpha$$

$$\alpha = \frac{d^2\theta}{dt^2}$$

$$-\frac{d}{2} mg\sin\theta = I_p \frac{d^2\theta}{dt^2}$$

$$\sin\theta \approx \theta$$

$$-\frac{d}{2} mg\theta = I_p \frac{d^2\theta}{dt^2}$$

$$\frac{-\frac{d}{2} mg\theta}{I_p} = \frac{d^2\theta}{dt^2}$$

$$0 = \frac{d^2\theta}{dt^2} + \frac{\frac{d}{2} mg\theta}{I_p}$$

$$\omega_0 \approx \sqrt{\frac{d/2\, mg}{I_p}}$$

$$\omega_0 = \sqrt{\frac{d/2\, mg}{1/3\, md^2}} = \sqrt{\frac{3/2\, g}{d}}$$

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{2d}{3g}}$$